

Kerr-Newman black holes with stationary charged scalar clouds

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Abstract

It is shown that Kerr-Newman black holes can support linear charged scalar fields in their exterior regions. To that end, we solve analytically the Klein-Gordon wave equation for a stationary charged massive scalar field in the background of a near-extremal Kerr-Newman black hole. In particular, we derive a simple analytical formula which describes the physical properties of these stationary bound-state resonances of the charged massive scalar fields in the Kerr-Newman black-hole spacetime.

I. INTRODUCTION

The ‘no-hair’ conjecture [1, 2] has played a central role in the physics of black holes since its introduction by Wheeler more than four decades ago [3–8]. This conjecture suggests that all asymptotically flat stationary black-hole spacetimes belong to one and the same family — the Kerr-Newman family of black holes [9–11]. If true, this conjecture implies that stationary black holes are characterized by only three externally observable physical parameters: mass, charge, and angular momentum.

The influential no-hair conjecture asserts, in particular, that static fields can not be supported in the spacetime region exterior to the black-hole horizon [1–8, 12]. It is therefore expected that external fields which are not associated with globally conserved charges would eventually be absorbed into the black hole or be radiated away to infinity. In accord with this physically motivated expectation, the dynamics of perturbation fields in a black-hole spacetime is characterized by quasinormal ringing, damped oscillations which reflect the gradual dissipation of energy from the black-hole exterior region [13]. At late times, these exponentially decaying oscillations are followed by inverse power-law decaying tails [14].

It is worth emphasizing that existing no-hair theorems [1–6] do not rule out the existence of non-static composed black-hole-field configurations. In particular, it has recently [15] been demonstrated explicitly that rotating Kerr black holes can support linearized stationary scalar configurations (scalar “clouds” [16, 17]) in their exterior regions. Since non-linear effects tend to stabilize hairy black-hole configurations [7, 8], we conjectured in [15] the existence of genuine non-static hairy black-hole-scalar-field configurations, which are the non-linear counterparts of the linear scalar clouds discussed in [15]. The existence of these non-static hairy black-hole-scalar-field configurations was demonstrated numerically most recently in the important work by Herdeiro and Radu [18]

The composed non-static black-hole-scalar-field configurations [19] studied in [15, 18] owe their existence to two distinct physical effects which together combine to trap the fields in the spacetime region exterior to the black-hole horizon:

(1) The first physical mechanism responsible for the existence of these non-static black-hole-bosonic-field configurations [15, 18] is the well-explored phenomenon of superradiant scattering of bosonic fields in black-hole spacetimes [20–24]. In particular, it is well established (see [25–27] and references therein) that a charged bosonic field of the form $e^{im\phi}e^{-i\omega t}$

interacting with a charged and rotating Kerr-Newman black hole can extract energy from the hole if the composed system is in the superradiant regime [25–27]

$$\omega < \omega_c , \quad (1)$$

where the critical frequency ω_c for superradiant scattering is given by [25–27]

$$\omega_c \equiv m\Omega_H + q\Phi_H . \quad (2)$$

Here [28]

$$\Omega_H = \frac{a}{r_+^2 + a^2} \quad \text{and} \quad \Phi_H = \frac{Qr_+}{r_+^2 + a^2} \quad (3)$$

are the angular velocity and the electric potential of the black hole, respectively (M, Q, Ma , and r_+ are respectively the mass, charge, angular momentum, and horizon-radius of the black hole). The parameters m and q in (2) are the azimuthal harmonic index and the charge coupling constant of the bosonic field, respectively.

(2) The second physical mechanism required for the existence of composed non-static black-hole-bosonic-field configurations [15, 18] is provided by the mutual gravitational attraction between the central black hole and the *massive* bosonic field. It is well known [23] that the mass term μ [see Eq. (7) below] of a massive field effectively acts as a reflecting wall which prevents low frequency modes with

$$\omega^2 < \mu^2 \quad (4)$$

from escaping to spatial infinity.

The main goal of the present study is to generalize the results of [15] to the regime of linearized charged massive scalar fields interacting with charged and rotating Kerr-Newman black holes. In particular, we shall show that Kerr-Newman black holes can support stationary charged scalar fields in their exterior regions. To that end, we shall solve analytically the Klein-Gordon wave equation for a stationary charged massive scalar field in the background of a near-extremal Kerr-Newman black hole. In particular, we shall derive a simple analytical formula which describes the physical properties of these stationary charged scalar clouds [16] in the Kerr-Newman black-hole spacetime.

II. DESCRIPTION OF THE SYSTEM

The physical system we consider consists of a test charged scalar field Ψ coupled to a Kerr-Newman black hole of mass M , angular-momentum Ma , and electric charge Q . In

Boyer-Lindquist coordinates (t, r, θ, ϕ) the spacetime metric is given by [9–11]

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a \sin^2 \theta d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2 \theta}{\rho^2}[adt - (r^2 + a^2)d\phi]^2, \quad (5)$$

where $\Delta \equiv r^2 - 2Mr + a^2 + Q^2$ and $\rho \equiv r^2 + a^2 \cos^2 \theta$. The black-hole (event and inner) horizons are located at the zeroes of Δ :

$$r_{\pm} = M \pm (M^2 - a^2 - Q^2)^{1/2}. \quad (6)$$

The dynamics of a charged massive scalar field Ψ in the Kerr-Newman spacetime is governed by the Klein-Gordon wave equation [25, 29]

$$[(\nabla^\nu - iqA^\nu)(\nabla_\nu - iqA_\nu) - \mu^2]\Psi = 0, \quad (7)$$

where μ and q are respectively the mass and charge coupling constant of the scalar field [30], and A_ν is the electromagnetic potential of the charged black hole. Substituting the decomposition [31]

$$\Psi = \sum_{l,m} e^{im\phi} S_{lm}(\theta; a\omega) R_{lm}(r; a, \omega) e^{-i\omega t}, \quad (8)$$

into the Klein-Gordon wave equation (7), one finds [25] that the radial function R_{lm} and the angular function S_{lm} can be determined from two coupled ordinary differential equations [see Eqs. (9) and (11) below] of the confluent Heun type [25, 29, 32–35].

The angular functions $S_{lm}(\theta; a\omega)$ are known as the spheroidal harmonics. These functions are solutions of the characteristic angular equation [25, 29, 32–35]

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{dS_{lm}}{d\theta} \right) + \left[K_{lm} + a^2(\mu^2 - \omega^2) - a^2(\mu^2 - \omega^2) \cos^2 \theta - \frac{m^2}{\sin^2 \theta} \right] S_{lm} = 0. \quad (9)$$

The spheroidal harmonics $S_{lm}(\theta; a\omega)$ determined from (9) should satisfy regularity boundary conditions at the poles $\theta = 0$ and $\theta = \pi$. These boundary conditions pick out a discrete set of eigenvalues $\{K_{lm}\}$ which are labeled by the integers l and m with $l \geq m$ [34]. In the regime $a^2(\mu^2 - \omega^2) \lesssim m^2$ [see Eqs. (19) and (30) below] one may treat the term $a^2(\omega^2 - \mu^2) \cos^2 \theta$ in (9) as a small perturbation to the familiar generalized Legendre equation [34]. This yields the perturbation expansion [34]

$$K_{lm} + a^2(\mu^2 - \omega^2) = l(l+1) + \sum_{k=1}^{\infty} c_k a^{2k} (\mu^2 - \omega^2)^k \quad (10)$$

for the angular eigenvalues (separation constants) K_{lm} . The expansion coefficients $\{c_k(l, m)\}$ are given in [34].

The radial Teukolsky equation is given by [25, 29]

$$\frac{d}{dr} \left(\Delta \frac{dR_{lm}}{dr} \right) + \left[\frac{H^2}{\Delta} + 2ma\omega - \mu^2(r^2 + a^2) - K_{lm} \right] R_{lm} = 0 , \quad (11)$$

where

$$H \equiv (r^2 + a^2)\omega - am - qQr . \quad (12)$$

Note that the angular eigenvalues $\{K_{lm}(a\omega)\}$ couple the radial Teukolsky equation (11) to the angular equation (9) [36].

In the present paper we shall analyze the bound-state resonances of the charged massive scalar fields in the Kerr-Newman black-hole spacetime. These exponentially decaying solutions are characterized by the asymptotic behavior [23]

$$R(r \rightarrow \infty) \sim \frac{1}{r} e^{-\sqrt{\mu^2 - \omega^2} r} . \quad (13)$$

Note that asymptotically decaying bound-states are characterized by $\omega^2 < \mu^2$ [see Eq. (4)]. In addition, we shall use the physically motivated boundary condition of purely ingoing waves (as measured by a comoving observer) at the outer horizon $r = r_+$ of the black hole [23]:

$$R(r \rightarrow r_+) \sim e^{-i(\omega - \omega_c)y} , \quad (14)$$

where ω_c is determined in (2) and the “tortoise” radial coordinate y is defined by $dy/dr = (r^2 + a^2)/\Delta$ [37].

The boundary conditions (13) and (14) single out a discrete family of complex frequencies $\{\omega(\mu, q, l, m, M, a, Q; n)\}$ [38] which correspond to the bound-state resonances of the charged massive scalar fields in the Kerr-Newman black-hole spacetime. The *stationary* bound-state resonances of the charged massive fields, which are the resonances we shall be interested in in this study, are characterized by $\Im\omega = 0$.

III. THE STATIONARY BOUND-STATE RESONANCES OF THE CHARGED MASSIVE FIELDS IN THE KERR-NEWMAN BLACK-HOLE SPACETIME

We shall now prove that, a charged scalar wave field with the critical frequency $\omega = \omega_c$ [the critical frequency for superradiant scattering, see Eq. (2)] corresponds to a stationary bound-state resonance of the charged massive scalar field in the Kerr-Newman black-hole spacetime. In particular, in this section we shall derive an analytical formula for the discrete spectrum of

scalar field masses, $\{\mu(q, l, m, M, a, Q; n)\}$ [38], which correspond to the stationary ($\Im\omega = 0$) bound-state resonances of the charged massive scalar fields.

It proves useful to define new dimensionless variables [25, 29]

$$x \equiv \frac{r - r_+}{r_+} \quad ; \quad \tau \equiv \frac{r_+ - r_-}{r_+} \quad ; \quad k \equiv 2\omega_c r_+ - qQ \quad , \quad (15)$$

in terms of which the radial Teukolsky equation (11) becomes

$$x(x + \tau) \frac{d^2 R}{dx^2} + (2x + \tau) \frac{dR}{dx} + VR = 0 \quad , \quad (16)$$

where $V \equiv H^2/r_+^2 x(x + \tau) - K + 2ma\omega_c - \mu^2[r_+^2(x + 1)^2 + a^2]$ and $H = r_+^2\omega_c x^2 + r_+ kx$. We shall henceforth consider near-extremal black holes with

$$\tau \ll 1. \quad (17)$$

We shall first study the radial equation (16) in the region $x \gg \tau$. In this far region the radial equation (16) is well approximated by

$$x^2 \frac{d^2 R}{dx^2} + 2x \frac{dR}{dx} + V_{\text{far}} R = 0 \quad , \quad (18)$$

where $V_{\text{far}} = (r_+\omega_c x + k)^2 - K + 2ma\omega_c - \mu^2[r_+^2(x + 1)^2 + a^2]$. Defining the dimensionless quantity

$$\epsilon \equiv \sqrt{\mu^2 - \omega_c^2} r_+ \quad , \quad (19)$$

one finds that the solution of Eq. (18) is given by [34, 39]:

$$R(x) = C_1 \times (2\epsilon)^{\frac{1}{2}+\beta} x^{-\frac{1}{2}+\beta} e^{-\epsilon x} M\left(\frac{1}{2} + \beta - \kappa, 1 + 2\beta, 2\epsilon x\right) + C_2 \times (\beta \rightarrow -\beta) \quad , \quad (20)$$

where $M(a, b, z)$ is the confluent hypergeometric function [34] and $\{C_1, C_2\}$ are normalization constants to be determined below. Here

$$\beta^2 \equiv K + \frac{1}{4} - 2ma\omega_c - k^2 + \mu^2(r_+^2 + a^2) \quad ; \quad \kappa \equiv \frac{\omega_c k - \mu^2 r_+}{\sqrt{\mu^2 - \omega_c^2}} \quad . \quad (21)$$

The notation $(\beta \rightarrow -\beta)$ in (20) means “replace β by $-\beta$ in the preceding term.” We shall henceforth consider the case of real β [40].

We shall next study the radial Teukolsky equation (16) in the region $x \ll 1$. The radial equation in this near-horizon region is given by Eq. (16) with $V \rightarrow V_{\text{near}} \equiv -K + 2ma\omega_c -$

$\mu^2(r_+^2 + a^2) + k^2 x/(x + \tau)$. The radial solution which satisfies the ingoing boundary condition (14) at the black-hole horizon is given by [34, 39]

$$R(x) = \left(\frac{x}{\tau} + 1\right)^{-ik} {}_2F_1\left(\frac{1}{2} + \beta - ik, \frac{1}{2} - \beta - ik; 1; -x/\tau\right), \quad (22)$$

where ${}_2F_1(a, b; c; z)$ is the hypergeometric function [34].

For near-extremal Kerr-Newman black holes with $\tau \ll 1$ [see Eq. (17)], there is an overlap region $\tau \ll x \ll 1$ in which the two radial expressions, (20) and (22), can be matched. The $x \ll 1$ limit of Eq. (20) is given by [34, 39]

$$R \rightarrow C_1 \times (2\epsilon)^{\frac{1}{2}+\beta} x^{-\frac{1}{2}+\beta} + C_2 \times (\beta \rightarrow -\beta), \quad (23)$$

and the $x \gg \tau$ limit of Eq. (22) is given by [34, 39]

$$R \rightarrow \tau^{\frac{1}{2}-\beta} \frac{\Gamma(2\beta)}{\Gamma(\frac{1}{2} + \beta - ik)\Gamma(\frac{1}{2} + \beta + ik)} x^{-\frac{1}{2}+\beta} + (\beta \rightarrow -\beta). \quad (24)$$

Matching the expressions (23) and (24) in the overlap region $\tau \ll x \ll 1$, one finds the two normalization constants $\{C_1, C_2\}$ of the radial function (20):

$$C_1(\beta) = \tau^{\frac{1}{2}-\beta} \frac{\Gamma(2\beta)}{\Gamma(\frac{1}{2} + \beta - ik)\Gamma(\frac{1}{2} + \beta + ik)} (2\epsilon)^{-\frac{1}{2}-\beta} \quad \text{and} \quad C_2(\beta) = C_1(-\beta). \quad (25)$$

The asymptotic $x \rightarrow \infty$ limit of the radial function (20) is given by [34, 39]

$$\begin{aligned} R(x \rightarrow \infty) \rightarrow & \left[C_1 \times (2\epsilon)^\kappa \frac{\Gamma(1+2\beta)}{\Gamma(\frac{1}{2} + \beta + \kappa)} x^{-1+\kappa} (-1)^{-\frac{1}{2}-\beta+\kappa} + C_2 \times (\beta \rightarrow -\beta) \right] e^{-\epsilon x} \\ & + \left[C_1 \times (2\epsilon)^{-\kappa} \frac{\Gamma(1+2\beta)}{\Gamma(\frac{1}{2} + \beta - \kappa)} x^{-1-\kappa} + C_2 \times (\beta \rightarrow -\beta) \right] e^{\epsilon x}. \end{aligned} \quad (26)$$

The bound-state resonances of the charged massive scalar fields are characterized by exponentially decaying radial solutions at spatial infinity [see Eq. (13)]. This implies that the coefficient of the growing exponent $e^{\epsilon x}$ in (26) must be zero:

$$C_1 \times (2\epsilon)^{-\kappa} \frac{\Gamma(1+2\beta)}{\Gamma(\frac{1}{2} + \beta - \kappa)} x^{-1-\kappa} + C_2 \times (\beta \rightarrow -\beta) = 0. \quad (27)$$

Substituting (25) into (27), one finds the characteristic resonance condition

$$\frac{1}{\Gamma(\frac{1}{2} + \beta - \kappa)} = \left[\frac{\Gamma(-2\beta)}{\Gamma(2\beta)} \right]^2 \frac{\Gamma(\frac{1}{2} + \beta - ik)\Gamma(\frac{1}{2} + \beta + ik)}{\Gamma(\frac{1}{2} - \beta - ik)\Gamma(\frac{1}{2} - \beta + ik)\Gamma(\frac{1}{2} - \beta - \kappa)} (2\epsilon\tau)^{2\beta} \quad (28)$$

for the stationary bound-state resonances of the charged massive scalar fields in the Kerr-Newman black-hole spacetime. Note that the r.h.s. of the resonance condition (28) is of

order $O[(\epsilon\tau)^{2\beta}] \ll 1$ [see Eq. (17) and Eq. (30) below]. Hence, one may use the well-known pole structure of the Gamma functions [34] in order to write the characteristic resonance condition (28) in the form

$$\frac{1}{2} + \beta - \kappa = -n + O[(\epsilon\tau)^{2\beta}] , \quad (29)$$

where $n = 0, 1, 2, \dots$ is the resonance parameter.

As we shall now show, the characteristic resonance condition (29) for the stationary bound-state resonances of the charged massive scalar fields in the Kerr-Newman black-hole spacetime can be solved *analytically* in the regime

$$\epsilon \ll 1 . \quad (30)$$

Taking cognizance of Eqs. (10), (15), (19), and (21), one finds [41, 42]

$$\beta = \beta_0 + O(\epsilon^2) \quad \text{and} \quad \kappa = \frac{\alpha}{\epsilon} + O(\epsilon) , \quad (31)$$

in the regime (30), where

$$\beta_0 \equiv \sqrt{\left(l + \frac{1}{2}\right)^2 - 2ma\omega_c - (2\omega_c r_+ - qQ)^2 + \omega_c^2(r_+^2 + a^2)} \quad ; \quad \alpha \equiv \omega_c r_+ (\omega_c r_+ - qQ) . \quad (32)$$

Substituting (31) into the resonance condition (29), one finds [43]

$$\epsilon(q, l, m, M, a, Q; n) = \frac{\alpha}{\beta_0 + \frac{1}{2} + n} . \quad (33)$$

Finally, we recall that the field-masses which correspond to the stationary bound-state resonances of the charged massive scalar fields are given by [see Eq. (19)]

$$\mu r_+(q, l, m, M, a, Q; n) = \sqrt{(\omega_c r_+)^2 + \epsilon^2} . \quad (34)$$

IV. WEAKLY BOUND STATIONARY RESONANCES: SOME PHYSICAL EXAMPLES

It is worth noting that the assumption $\epsilon \ll 1$ [see Eq. (30)] corresponds to weakly bound-state resonances ($\omega_c \lesssim \mu$) of the charged massive scalar fields in the Kerr-Newman black-hole spacetime. As we shall now show, this assumption is satisfied in several distinct physical regimes:

(1) Slowly rotating Kerr-Newman black holes with $a \ll M$. In this case one finds [see Eqs. (2) and (3)]

$$\omega_c r_+ = qQ + m\bar{a} - qQ\bar{a}^2 + O(\bar{a}^3) , \quad (35)$$

where

$$\bar{a} \equiv \frac{a}{r_+} . \quad (36)$$

Substituting (35) into (32), one obtains

$$\beta_0 = \left(l + \frac{1}{2}\right)[1 + O(\bar{a})] \quad ; \quad \alpha = qQm\bar{a}[1 + O(\bar{a})] , \quad (37)$$

which implies [see Eq. (33)]

$$\epsilon = \frac{qQm}{l+1+n} \cdot \bar{a}[1 + O(\bar{a})] \ll 1 . \quad (38)$$

Substituting (35) and (38) into (34), one finds

$$\mu r_+ = qQ \left\{ 1 + \frac{m}{qQ} \cdot \bar{a} + \left[\frac{1}{2} \left(\frac{m}{l+1+n} \right)^2 - 1 \right] \bar{a}^2 + O(\bar{a}^3) \right\} \quad (39)$$

for the field-masses of the stationary bound-state resonances.

(2) Composed Kerr-Newman-scalar-field configurations in the regime [44]

$$\omega_c r_+ \ll 1 \quad \text{with} \quad qQ = O(1). \quad (40)$$

Substituting (40) into (32), one finds

$$\beta_0 = \sqrt{\left(l + \frac{1}{2}\right)^2 - (qQ)^2} + O(\omega_c r_+) \quad \alpha = -qQ\omega_c r_+[1 + O(\omega_c r_+)] , \quad (41)$$

which implies [see Eq. (33)]

$$\epsilon = -\frac{2qQ}{\sqrt{(2l+1)^2 - (2qQ)^2} + 1 + 2n} \cdot \omega_c r_+[1 + O(\omega_c r_+)] \ll 1 . \quad (42)$$

Substituting (42) into (34), one finds

$$\mu r_+ = \omega_c r_+ \sqrt{1 + \frac{(2qQ)^2}{[\sqrt{(2l+1)^2 - (2qQ)^2} + 1 + 2n]^2}} [1 + O(\omega_c r_+)] . \quad (43)$$

(3) Composed Kerr-Newman-scalar-field configurations in the regime [45]

$$\omega_c r_+ = qQ + \delta \quad \text{with} \quad \delta \ll 1 . \quad (44)$$

Substituting (44) into (32), one finds

$$\beta_0 = \sqrt{\left(l + \frac{1}{2}\right)^2 - m^2} + O(\delta) \quad ; \quad \alpha = qQ\delta[1 + O(\delta)] , \quad (45)$$

which implies [see Eq. (33)]

$$\epsilon = \frac{2qQ}{\sqrt{(2l+1)^2 - 4m^2} + 1 + 2n} \cdot \delta[1 + O(\delta)] \ll 1 . \quad (46)$$

Substituting (44) and (46) into (34), one finds

$$\mu r_+ = qQ \left\{ 1 + \frac{1}{qQ} \cdot \delta + \frac{2}{[\sqrt{(2l+1)^2 - 4m^2} + 1 + 2n]^2} \cdot \delta^2 + O(\delta^3) \right\} . \quad (47)$$

(4) Nearly polar scalar clouds with $l \gg \max(m, qQ)$. In this case one finds [see Eqs. (32) and (33)]

$$\epsilon = \frac{\alpha}{l} [1 + O(l^{-1})] \ll 1 . \quad (48)$$

Substituting (48) into (34), one finds

$$\mu r_+ = \omega_c r_+ + \frac{\alpha^2}{2\omega_c r_+} \cdot \frac{1}{l^2} [1 + O(l^{-1})] . \quad (49)$$

(5) High overtone modes with $n \gg \max(l, qQ)$. In this case one finds [see Eqs. (32) and (33)]

$$\epsilon = \frac{\alpha}{n} [1 + O(n^{-1})] \ll 1 . \quad (50)$$

Substituting (50) into (34), one finds

$$\mu r_+ = \omega_c r_+ + \frac{\alpha^2}{2\omega_c r_+} \cdot \frac{1}{n^2} [1 + O(n^{-1})] . \quad (51)$$

V. SUMMARY AND DISCUSSION

In summary, we have shown that Kerr-Newman black holes can support linear charged scalar fields in their exterior regions. To that end, we have solved analytically the Klein-Gordon wave equation for a stationary charged massive scalar field in the background of a near-extremal Kerr-Newman black hole. In particular, we have derived an analytical formula [see Eqs. (32)-(34)] which determines the discrete spectrum of scalar field masses,

$\{\mu(q, l, m, M, a, Q; n)\}$, which correspond to the stationary bound-state resonances of the charged massive scalar fields in the Kerr-Newman black-hole spacetime.

It is worth emphasizing that the stationary charged scalar clouds studied in this paper owe their existence to the rotation of the central black hole. In particular, we have shown that there are no stationary bound-state configurations of the charged scalar field with $ma = 0$ [41]. This conclusion is in accord with the results presented in [18, 42] for static charged Reissner-Nordström black holes [46].

Finally, we would like to note that, it would be physically interesting to generalize our *analytical* results for the linear charged scalar clouds to the non-linear regime of a genuine charged scalar hair. Such a generalization would probably require a fully non-linear *numerical* [18] analysis of the combined Einstein-Maxwell-Klein-Gordon equations.

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